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17MAT41

Fourth Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note:1) ***Answer any FIVE full questions, choosing ONE full question from each module.***
2) ***Use of Statistical tables allowed.***

Module-1

1 a. Use Taylor's series to obtain approximate value of y at x = 0.1 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$. **(06 Marks)**

b. Apply Runge Kutta method of fourth order to find an approximate value of y when x = 0.2 for the equation $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ taking h = 0.2. **(07 Marks)**

c. Using Milne's predictor – corrector method, find y when x = 0.8 given $\frac{dy}{dx} = x - y^2$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. **(07 Marks)**

OR

2 a. Given that $\frac{dy}{dx} = \log(x + y)$ and y(1) = 2, then find y(1.2) in step of 0.2 using modified Euler's method carry out two iterations. **(06 Marks)**

b. Using fourth order Runge-Kutta method to find y at x = 0.2 equation given that $\frac{dy}{dx} = x + y$, y(0) = 1 and h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1 + y)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adam's-Bashforth predictor-corrector method. **(07 Marks)**

Module-2

3 a. Using Runge-Kutta method, solve $\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y^2$ for x = 0.2, correct to three decimal places, with initial conditions $y(0) = 1$, $y'(0) = 0$. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, then $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Express $f(x) = 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(07 Marks)**

OR

4 a. Apply Milne's predictor-corrector method to compute y(0.4) given the differential equation $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following initial values:
$y(0) = 1, y(0.1) = 1.1103, y(0.2) = 1.2427, y(0.3) = 1.399$
$y'(0) = 1,\ y'(0.1) = 1.2103,\ y'(0.2) = 1.4427,\ y'(0.3) = 1.699$ **(06 Marks)**

b. With usual notation, show that

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$ **(07 Marks)**

c. With usual notation, derive the Rodrigue's formula $P_n(x) = \frac{1}{(2^n)n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$. **(07 Marks)**

Module-3

5 a. Find the bilinear transformation which map the points z = 0, 1, ∞ into the points $w = -5, -1, 3$ respectively. **(06 Marks)**

b. Derive Cauchy-Riemann equations in Cartesian form. **(07 Marks)**

c. Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)}dz$ where $C: |z| = 2.5$ by residue theorem. **(07 Marks)**

OR

6 a. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(06 Marks)**

b. Discuss the transformation $W = Z^2$. **(07 Marks)**

c. Evaluate $\int_C \frac{e^{2z}}{(z+1)(z+2)}$, where C is the circle $|z| = 3$, using Cauchy residue theorem. **(07 Marks)**

Module-4

7 a. The probability density function of a variate x given by the following table:

X	−3	−2	−1	0	1	2	3
P(X)	K	2K	3K	4K	3K	2K	K

Find the value of K, mean and variance. **(06 Marks)**

b. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for, (i) more than 2150 hours, (ii) less than 1950 hours, (iii) more than 1920 hours and but less than 2160 hours.
Given : $A(0 < z < 1.83) = 0.4664$, $A(0 < z < 1.33) = 0.4082$ and $A(0 < z < 2) = 0.4772$ **(07 Marks)**

c. A joint probability distribution is given by the following table:

X \ Y	−3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Determine the marginal probability distributions of X and Y. Also find COV(X, Y). **(07 Marks)**

OR

8 a. Derive mean and variance of the Poisson distribution. **(06 Marks)**

b. In a certain town the duration of a shower is exponentially distributed within mean 5 minute. What is the probability that a shower will last for,
(i) less than 10 minutes (ii) 10 minutes or more
(iii) between 10 and 12 minutes. **(07 Marks)**

c. Given,

X \ Y	0	1	2	3
0	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	0

(i) Find Marginal distribution of X and Y.
(ii) Find E(X), E(Y) and E(XY). **(07 Marks)**

Module-5

9 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. **(06 Marks)**

b. Five dice were thrown 96 times and number 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as follows:

No. of dice showing 1, 2 or 3 :	5	4	3	2	1	0
Frequency :	7	19	35	24	8	3

Test the hypothesis that the data follow a binomial distribution at 5% level of significance $\left(\chi^2_{0.05} = 11.07 \text{ for d.f is } 5\right)$. **(07 Marks)**

c. A student's study habits are as follows:
If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night he is 60% sure not to study the next night. In the long run how often does he study? **(07 Marks)**

OR

10 a. If $p = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$, find the fixed probabilities vector. **(06 Marks)**

b. A random sample of 10 boys had the following I.Q's : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this supports the hypothesis that the population mean of I.Q's is 100 at 5% level of significance? ($t_{0.05} = 2.262$ for 9 d.f) **(07 Marks)**

c. Explain : (i) Transient state (ii) Absorbing state (iii) Recurrent state. **(07 Marks)**

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